Exotic Matter in Compact Stars
Limits and Consequences

OUTLINE

- observational constraints
- some remarks on nuclear structure
- hadrons and quarks
- susceptibilities
- adding magnetic fields
- to-do list

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neutron stars are remnants of Type II supernovae

1 to 2 solar masses, radii around 10 - 15 km
maximum central densities $4$ to $10 \rho_0$

about 2000 known neutron stars

hyper star

hybrid star
Masses of Neutron Stars

Masses of radio pulsars

Kiziltan, Kottas, Thorsett, astro-ph:1011.4291

no signature for mass cut off

\[ M = (1.97 \pm 0.04) \, M_\odot \]


well established - heavy neutron stars

Lattimer, Prakash, astro-ph:1012.3208

Masses of Neutron Stars

\[ M = (2.4 \pm 0.12) \, M_\odot \]


current benchmark for NS models

\[ M = (1.97 \pm 0.04) \, M_\odot \]


new observation PSR J0348+0432

\[ M = (2.01 \pm 0.04) \, M_\odot \]

the usual phase diagram (sketch) of strong interactions

Practical model useful for heavy-ion simulations and compact star physics

correct asymptotic degrees of freedom
reasonable description on a quantitative level for high T down to nuclei
possibility of studying first-order as well as cross-over transitions

connect both worlds in some reasonable way
hadronic SU(3) approach based on non-linear realization of extended $\sigma\omega$ model

Lowest multiplets

$$B = \{ \rho, n, \Lambda, \Sigma^{\pm/0}, \Xi^{-/0} \} \text{ baryons}$$

$$\text{diag} \ (V) = \{ (\omega + \rho)/\sqrt{2}, (\omega - \rho)/\sqrt{2}, \phi \} \text{ vector mesons}$$

$$\text{diag} \ (X) = \{ (\sigma + \delta)/\sqrt{2}, (\sigma - \delta)/\sqrt{2}, \zeta \} \text{ scalar mesons}$$

Mean fields generate scalar attraction and vector repulsion

Scalar self interaction

$$L_0 = -\frac{1}{2} k_0 I_2 + k_1 (I_2)^2 + k_2 I_4 + 2 k_3 I_3 + L_{ESB}$$

invariants

$$I_1 = \text{Tr}(X) \quad I_2 = \text{Tr}(X)^2 \quad I_3 = \text{det}(X)$$

+ dilaton field

$$L_{\chi} = -k_4 \chi^4 - \frac{1}{4} \chi^4 \ln \left(\frac{\chi^4}{\chi_0^4}\right) + \delta/3 \chi^4 \ln \left(I_3/<X>\right)$$
hadronic SU(3) approach ... continued

\[ L_{BW} = - \sqrt{2} g_8^W (\alpha_w [\overline{BOBW}]_F + (1 - \alpha_w) [\overline{BOBW}]_D) \]

- \( g_1^W / \sqrt{3} \) Tr(\overline{BOB}) Tr (W)

\[ \text{SU}(3) \text{ interaction} \]

\[ V(M) \quad \langle \sigma \rangle = \sigma_0 \neq 0 \quad \langle \zeta \rangle = \zeta_0 \neq 0 \]

\[ \sigma \sim \langle \overline{u} u + \overline{d} d \rangle \quad \zeta \sim \langle \overline{s} s \rangle \quad \delta^0 \sim \langle \overline{u} u - \overline{d} d \rangle \]

explicit breaking \( \sim \) Tr [ c \sigma ] \( (\sim m_q \overline{q} q) \)

fix scalar parameters to

baryon masses, decay constants, meson masses
binding energy $E/A \sim -16.2$ MeV  saturation $(\rho_B)_0 \sim 0.16/fm^3$

compressibility $\sim 235$ MeV  asymmetry energy $\sim 32.9$ MeV

slope L $\sim 66.7$ MeV

1d to 3d code  deformed calculation of all measured ($\sim 800$) even-even nuclei

error in energy $\varepsilon (A > 50) \sim 0.17\%$  (NL3: 0.25\%)
$\varepsilon (A > 100) \sim 0.12\%$  (NL3: 0.16\%)

+ correct binding energies of hypernuclei

new fit for large star masses
by T. Schürhoff  $\varepsilon \sim 0.28$, $\kappa < 300$ MeV, $M \sim 2 M_\odot$

stellar crust calculations in progress
Nobelium (Z=102) isotopes

Experimental values:
- $\beta_2 \sim 0.32 \pm 0.02 (A=254)$
- $0.31 \pm 0.02 (A=252)$
medium-heavy nucleus $^{68}$Se

Z = 32

experiment - oblate groundstate around $\beta_2 \sim -0.3$
+ strongly prolate excited band

Sulfur isotopes
measured deformations
Compared to calculation
Neutron star masses including different sets of particles

Tolman-Oppenheimer-Volkov equations, static spherical star changing masses with degrees of freedom

large star masses even with spin 3/2 resonances

Impact of $\Phi$ field

rescale $g_{B\Phi}$ coupling parameters, $f_s$(core) varies between 0.1 and 1

$M_{\text{max}} [M_\odot]$ 

2

Nijmegen, no YY interaction

$f_s = n_s / n_B$

Schulze, Rijken, PRC 84, 035801 (2011)
Include modified distribution functions for quarks/antiquarks

\[ \Omega_q = -T \sum_{j \in Q} \frac{\gamma_i}{(2\pi)^3} \int d^3k \ln \left( 1 + \Phi \exp \frac{E^*_j - \mu_i}{T} \right) \]

\[ \Phi \quad \text{confinement order parameter}^* \]

Following the parametrization used in PNJL calculations

\[ U = -\frac{1}{2} a(T) \Phi \Phi^* + b(T) \ln[1 - 6 \Phi \Phi^* + 4 (\Phi \Phi^*)^3 - 3 (\Phi \Phi^*)^2] \]

\[ a(T) = a_0 T^4 + a_1 T^0 T^3 + a_2 T^2 T^2, \quad b(T) = b_3 T^3 T \]

The switch between the degrees of freedom is triggered by excluded volume corrections

thermodynamically consistent - 

\[ V_q = 0 \]

\[ V_h = v \]

\[ \tilde{\mu}_i = \mu_i - v_i P \]

\[ e = \tilde{e} / (1 + \Sigma v_i \tilde{p}_i) \]

no reconfinement!

equation of state stays causal!

Steinheimer, SWS, Stöcker JGP 38, 035001 (2011)
Order parameters for chiral symmetry and confinement in $\mu$ and $T$

except for liquid-gas no first-order transition
results for hot matter at vanishing chemical potential

points are various lattice results

Polyakov loop

Interaction measure

scalar condensate

densities
Initial transverse distributions of the deconfined fraction for central Au+Au collisions at different beam energies.

\[ E_{\text{lab}} = \begin{cases} 10\text{A GeV} \\ 40\text{A GeV} \\ 160\text{A GeV} \end{cases} \]

- Part of UrQMD hybrid transport code

Simple time evolution of \( f_s \) including \( \pi, K \) evaporation (\( E/A = 40 \text{ GeV} \))

\[ f_s = \frac{n_s}{n_B} \]

C. Greiner et al., PRD38, 2797 (1988)
Mass-radius relation and structure

Hybrid star within the QH model and realistic ground state + nuclei

different quark-vector couplings

$g_{q\omega} \sim 1/6 \, g_{n\omega}$

$g_{q\omega} \sim 1/3 \, g_{n\omega}$

$M_{\text{max}} = 2.23 \, M_\odot$
$R = 11.2 \, \text{km}$

$\rho > 3 \, \rho_0 :$
$c_s^2 \sim -0.1 + \rho \, \text{fm}^3$
normalized particle numbers in hybrid star

1\textsuperscript{st} order phase transition due to strangeness

play around with parameters second family of compact stars
M-R diagram in QH model

- Maxwell / Gibbs construction for local / global charge neutrality
- Large mixed region
- Baryonic star with a 2km hybrid core

Potential fitted to lattice data generate critical end point

\[ U = - \frac{1}{2} a(T, \mu) \Phi \Phi^* + \ldots \]

\[ a(T, \mu) = a_0 T^4 + a_1 \mu^4 + a_2 \mu^2 T^2 \]

(Also Schäfer et al, PRD 76 074023)
Hybrid Stars, Quark Interactions

ingredients –
Standard baryonic EOS (G300) plus MIT bag model + $\alpha_s$ corrections

Fast cooling in the quark core
need gaps in the quark phase

baryons alone $M_{\text{max}} \sim 1.8 M_{\text{solar}}$

no $\alpha_s$
Susceptibility $c_2$ in PNJL and QH model for different quark vector interactions

$$P(T,\mu) = P(T) + c_2(T) \mu^2 T^2 + \ldots$$

Small quark vector repulsion !!

PNJL

QH

$$g_{q\omega} = 0$$

$$g_{q\omega} = g_{n\omega} / 3$$

Steinheimer, SWS, PLB 696, 257 (2011)
analogous behaviour of strange susceptibility

\[ X_s = T^2 \frac{d^2( P/T^4 )}{(d \mu_S)^2} \bigg|_{\mu_B, \mu_S = 0} \]
observed surface fields up to $\sim 10^{15}$ G - magnetars

might be significantly larger in the interior of the star

Landau levels:

$$E_{i\nu s}^* = \sqrt{k_{z_i}^2 + \left(\sqrt{m_i^*} + 2\nu q_i |\vec{B}^* - s_i \kappa_i B^*| - B_{\nu s}^*\right)^2}$$

anomalous magnetic moment

simple parameterization of the field

$$B^*(\mu_B) = B_{\text{surf}} + B_c \left[1 - e^{\left(\frac{\mu_B - 938}{938}\right)^a}\right]$$

Dexheimer, Negreiros, SWS EPJA 2012
Polyakov loop and 1\textsuperscript{st} order transition

\begin{equation}
e B_{cr} = m_e^2, \quad B_{cr} = 4.4 \times 10^{13} \text{ G}
\end{equation}

\begin{equation}
(1.5 \times 10^{20} \text{ G})
\end{equation}

Particle densities \( B_c = 7.2 \times 10^{18} \text{ G} \)

PT gets shifted somewhat to higher \( \mu \)

\textit{Dexheimer, Negreiros, SWS} EPJA 2012
The energy-momentum tensor is not isotropic, and consistent modeling of the star is needed. The tensor is given by:

\[ T^{\mu \nu} = \text{diag}(B^2, B^2, B^2, -B^2) / (8\pi^2) \]

This requires adding isotropic pressure for "interesting" field strengths where field energy dominates changes in the EOS. The impact on the M(R) diagram for neutron/hybrid star is shown in the graph.

2d calculations:
Cardall et al. APJ 554, 322 (2001)
include effect of magnetic field in GR

assume a dipole field

expand metric into multipoles

analogous to Hartle/rotation

\[ \epsilon = \epsilon_m + \frac{B^2}{8\pi} \]
\[ P_\perp = P_m + \frac{B^2}{8\pi} \]
\[ P_\parallel = P_m - \frac{B^2}{8\pi}. \]

\[ P = P_m + \frac{B^2}{8\pi} (1 - 2\cos^2 \theta) \]

\[ P = P_m + [p_0 + p_2 P_2(\cos \theta)] \]

\[ ds^2 = -e^\nu(r)[1 + 2(h_0(r) + h_2(r)P_2(\cos \theta))]dt^2 \]
\[ +e^\lambda(r)[1 + \frac{e^\lambda(r)}{r}(m_0(r) + m_2(r)P_2(\cos \theta))]dr^2 \]
\[ +r^2[1 + 2k_2(r)P_2(\cos \theta)](d\theta^2 + \sin^2 \theta d\phi^2), \]

Hartle, APJ, 1967
Mass shift and deformation for different values of central energy density and field

NW, TM1  hard / soft equation of state

assume a profile of the magnetic field

\[ B(n_b) = B_s + B_0 \left\{ 1 - e^{-\alpha\left(\frac{n_b}{n_0}\right)^\gamma} \right\} \]

Corrections to pressure ~ original value

eccentricity  =  \( (1 - \frac{R_p^2}{R_e^2})^{1/2} \)

Mallick, SWS arxiv:1307.5184
Conclusions, Outlook

• heavy hyper stars possible - not too hyper, though
• formulation of realistic quark-hadron model possible
• 2 solar mass hybrid star - again not very strange
• serious conflicts with lattice QCD
• simple treatment of deformation for high magnetic fields

• comprehensive equation of state for a wide range of densities/temperatures (supernovae, mergers)

Many thanks to the organizers!
Kaon energies as function of density for neutron star at \( T = 0 \)

- Kaon condensation sets in at around 5.5 \( \rho_0 \)
- No significant change of star properties

\[ U_{K^-}(\rho_0) \sim -50 \text{ MeV} \]

Correct \( a_{KN} \) values

Hyperons shift \( \rho_c \) to higher values

Mishra, Kumar, Sanyal, SWS, EPJA 41, 205
polar and equatorial radii

$B_c = 10^{19} \text{ G}$
UrQMD/Hydro hybrid simulation of a Pb-Pb collision at 40 GeV/A
EOS part of the package

red regions show the areas dominated by quarks
Extension of the parity model to SU(3)

Baryon SU(3) multiplet + parity doublets

Similar approach, SU(3)-invariant potential for scalar fields

\[ E_\pm = \sqrt{(g_1 \sigma + g_2 \varsigma)^2 + m_0^2} \pm (g_1' \sigma + g_2' \varsigma) \]

simplify investigation – same mass shift for whole octet

Candidates – \( \Lambda(1670), \Sigma(1750), \Xi (?) \) overall unclear

Steinheimer, SWS, Stöcker, JPhysG 38, 035001 (2011)

first study - Nemoto et al. PRD 57, 4124 (1998)
First order transition for masses ≥ 1470 MeV, below crossover
eccentricity = \left( 1 - \frac{R_p^2}{R_e^2} \right)^{1/2}

Mallick, SWS arxiv:1307.5184

 Corrections to pressure ~ original value

fixed central density, varying the magnetic field
Excited quark-hadron matter in the parity-doublet approach

Liquid-gas phase transition

Chiral transition

2 different values for $T_0$

$\sigma/\sigma_0 = 0.2$

$\Phi = 0.8$