Euclidean Scalar Field with Quenched Additive Anisotropic Disorder: An Analog Model for Euclidean Wormholes Effects

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- In the classical general theory of relativity, the spacetime is a smooth pseudo-Riemannian manifold and the gravitational field is described as the curvature in the spacetime. It is a local theory where all physical quantities formulated in a general way are related to local dynamical equations.
- Quantum field theory was initially defined in the four dimensional Minkowski space with a causal and metric structure. The symmetry group is the Poincaré group generated by the Lorentz transformations and translations.
- In the canonical quantization procedure, quantum fields are operator values distributions $\phi(f)$, *i.e.*, must be smeared out with smooth compacted support test functions. These smeared fields act as unbounded operators in a Hilbert space.

- In the construction of the formalism the principles underlying a quantum field theory are the probabilistic interpretation of expectation values, this special relativity and locality. In local quantum field theory causality is manifested as the requirement for the smeared fields $\phi(f)$ and $\phi(g)$ commute when the supports of f and g are space-like separated.
- The program of describe the gravitational field using quantum theory introduce many conceptual difficulties as, for examples, the causal structure and locality issues.
- Quantum field theory on gravitational background space-time is an intermediate step toward to our understanding of the program of quantization of the gravitational field. In this approach, quantum fields are defined on classical space-times.
- To go further, one can discuss the effects of the fluctuations of the metric fields over the quantum matter fields. One can show that the effects of a bath of gravitons in a squeezed state is to fluctuates the light cones.

- Unruh has shown that the propagation of sound waves in a hypersonic fluid is equivalent to the propagation of scalar waves in black-hole spacetime.
- Quantizing the acoustic wave in such a physical system with a sonic horizon implies that the sonic black-hole can emit sound waves with a thermal spectrum. Analog model for the Hawking result: the presence of phononic Hawking radiation from the acoustic horizon.
- An analog model for quantum gravity effects in condensed matter was proposed. The situation discussed was of phonons propagating in a fluid with a random velocity wave equation.
- Quantum gravity effects in the laboratory: entangled non-gravitational systems may exhibit phenomena characteristic of quantum gravity.
- Here we discuss an analog model for Euclidean wormholes effects. We discuss an Euclidean scalar field theory with quenched additive anisotropic disorder.

- In classical statistical mechanics of Hamiltonian systems, any state is a probability measure on the phase space. The expectation value of any observable can be obtained from an average constructed with the Gibbs measure

$$d\mu_{\text{Gibbs}} = \frac{1}{Z} e^{-\beta H} d\mu_{\text{Liouville}}, \tag{1}$$

where Z the partition function, H is the Hamiltonian, $\beta = 1/T$, T is the absolute temperature and $d\mu_{\text{Liouville}}$ is the Liouville measure.

- The partition function is obtained from a normalization procedure.
- For systems described in the continuum with uncountable infinite degrees of freedom, this framework can be maintained, with a formal Lebesgue measure.
- Euclidean functional methods, with functional of probability measures, introduced classical probabilistic concepts in quantum field theory.

- The analytic continuation of vacuum expectation values for imaginary time of the Wightman functions are the Schwinger functions. These functions have a probabilistic interpretation as correlations of a random field.
- The Gaussian free field: a Gaussian random distribution with covariance given by the Green function of the Laplace operator in $\mathbb{R}^d$, i.e.,

$$(-\Delta + m^2)G_0(x,y) = \delta^d(x-y). \tag{2}$$

- For a scalar field defined in $\mathbb{R}^d$, the moments of a probability measure are the n-point correlation functions given by

$$\langle\varphi(x_1)..\varphi(x_k)\rangle = \frac{1}{Z}\int [d\varphi]\prod_{i=1}^{k}\varphi(x_i)\exp\left(-S(\varphi)\right). \tag{3}$$

The $[d\varphi]$ is a functional measure, *i.e.*, a measure in the space of all field configurations and $S(\varphi)$ is the Euclidean action of the system.

The random field Ising model in a hypercubic lattice in d-dimensions is defined by the Hamiltonian

$$H = -J\sum_{(i,j)}^{N} \sigma_i\sigma_j - \sum_i h_i\sigma_i, \tag{4}$$

where (i, j) indicates that the sum is performed over nearest neighbour pairs, $\sigma_i = \pm 1$ and h_i is a random variable. The Edwards Anderson Hamiltonian is

$$H = -\sum_{<i,j>} J_{ij}\sigma_i\sigma_j - h\sum_i \sigma_i. \tag{5}$$

With a quenched disorder field η, one has to compute the disorder-averaged generating functional of connected correlations functions:

$$W(j) = \int [d\,\eta]P(\eta)\ \ln Z(\eta, j), \tag{6}$$

where $[d\,\eta]$ is a functional measure and $P(\eta)$ is the probability distribution of the disorder.

- The fundamental question is whether there exists a thermodynamic spin glass phase: a transition from the high temperature paramagnetic state to a low temperature spin glass state.
- We construct $Z^k = Z \times Z \times ... \times Z$. We interpret Z^k as the partition function of a system formed with k statistically independent copies of the original system.
- The replica method is used to average over the disorder.

$$F = -\lim_{n\to 0}\left[\frac{\mathbb{E}[Z^n]-1}{n}\right]. \tag{7}$$

- "The replica theory puts the dirty under the carpet and cleaning under the carpet is not an easy job" (Parisi).
- "A strong effort must be done to decode the replica theory and to give a physical meaning to the results of the replica approach" (Parisi).

Let us discuss the spectral zeta function regularization. If λ_k is a sequence of non-zero complex numbers, we define the zeta regularized product of these numbers as

$$\prod_k \lambda_k = e^{-\frac{d}{ds}\zeta(s)|_{s=0}}, \tag{8}$$

where

$$\zeta(s) = \sum_k \frac{1}{\lambda_k^s} \;\; for \;\; Re(s_0) > 0, \tag{9}$$

is the zeta function associated with the sequence λ_k. If λ_k is the sequence of the positive eingenvalues of the Laplacian, then the zeta regularized product is the determinant of the Laplacian. The free energy of a system with these eigenvalues is

$$F = \frac{1}{2}\frac{d}{ds}\zeta_D(s)|_{s=0}. \tag{10}$$

The next step is to show how to generalize the usual zeta functions to a broader context.
Recall that a measure space $(\Omega, \mathcal{W}, \eta)$ consists in a set Ω, a σ-algebra $\mathcal{W}$ in Ω, and a measure η on this σ-algebra. Given a measure space $(\Omega, \mathcal{W}, \eta)$ and a measurable $f : \Omega \to (0, \infty)$, we define the associated generalized ζ-function as

$$\zeta_{\eta,f}(s) = \int_{\Omega} f(\omega)^{-s}\, d\eta(\omega) \tag{11}$$

for those $s \in \mathbb{C}$ such that $f^{-s} \in L^1(\eta)$, where in the above integral

$$f^{-s} = \exp(-s \log(f)) \tag{12}$$

is obtained using the principal branch of the logarithm.

Let us assume a $\lambda\varphi^4 + \rho\varphi^6$ scalar model without disorder. The partition function is defined as

$$Z = \int [d\varphi]\, \exp\big(-H(\varphi)\big), \tag{13}$$

where $H(\varphi) = H_0(\varphi) + H_I(\varphi)$. We have

$$H_0(\varphi) = \int d^dx\, \frac{1}{2}\varphi(x)\Big(-\Delta + m_0^2\Big)\varphi(x), \tag{14}$$

where Δ is the Laplacian in $\mathbb{R}^d$. $H_I(\varphi)$ is defined by

$$H_I(\varphi) = \int d^dx\, \left(\frac{\lambda_0}{4}\varphi^4(x) + \frac{\rho_0}{6}\varphi^6(x)\right). \tag{15}$$

Here $[d\varphi]$ is the functional measure, or a formal Lebesgue measure given by $[d\varphi] = \prod_x d\varphi(x)$.

A multiplicative quenched disorder field defines the Hamiltonian

$$H_0(\varphi,\delta m_0^2) = \frac{1}{2}\int d^dx\,\varphi(x)\Big(-\Delta + m_0^2 - \delta m_0^2(x)\Big)\varphi(x). \qquad (16)$$

Defining $[d\delta m_0^2]$ as a functional measure, the probability distribution of the disorder is $[d\delta m_0^2]P\left(\delta m_0^2\right)$ where

$$P(\delta m_0^2) = p_0 \exp\left(-\frac{1}{4\sigma^2}\int d^dx\,\left(\delta m_0^2(x)\right)^2\right). \qquad (17)$$

The quantity σ is a small parameter that describes the strength of disorder and p_0 is a normalization constant. In this case we have a delta correlated disorder field since

$$\mathbb{E}[\delta m_0^2(x)\delta m_0^2(y)] = \sigma^2\delta^d(x-y). \qquad (18)$$

Our aim is to compute the disorder-averaged free energy given by

$$F = -\frac{1}{\beta}\int [d\,\delta m_0^2] P(\delta m_0^2)\ \ln Z(\delta m_0^2), \tag{19}$$

where again $[d\,\delta m_0^2]$ is also a functional measure. Here we use the definition of the distributional zeta-function $\Phi(s)$, inspired in the spectral zeta-function, as

$$\Phi(s) = \int [d\delta m_0^2] P(\delta m_0^2)\,\frac{1}{Z(\delta m_0^2)^s}, \tag{20}$$

for $s \in \mathbb{C}$, this function being defined in the region where the above integral converges. The average free energy can be written as

$$F = \frac{1}{\beta}(d/ds)\Phi(s)|_{s\to 0^+}, \qquad Re(s) \geq 0, \tag{21}$$

where $\Phi(s)$ is well defined.

Hence, using analytic tools, and integrating over the disorder, the average free energy can be represented by

$$F = \frac{1}{\beta}\left[\sum_{k=1}^{\infty}\frac{(-1)^k a^k}{k!\,k}\mathbb{E}\left[Z^k\right] + \log a + \gamma + R(a)\right]. \tag{22}$$

$$R(a) = \int [d\delta m_0^2] P(\delta m_0^2) \int_a^{\infty} \frac{dt}{t}\, e^{-Z(\delta m_0^2)t}. \tag{23}$$

Using the probability distribution for the disorder and the Hamiltonian of the model, this quantity is given by

$$\mathbb{E}\left[Z^k\right] = \int \prod_{i=1}^{k}\left[d\varphi_i^{(k)}\right] e^{-H_{eff}\left(\varphi_i^{(k)}\right)}. \tag{24}$$

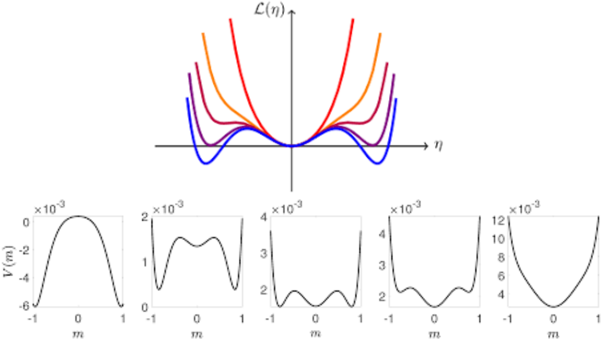
The effective Hamiltonian, $H_{eff}(\varphi_i^{(k)})$ is written as

$$H_{eff}\left(\varphi_i^{(k)}\right) = \int d^d x \left[\frac{1}{2}\sum_{i=1}^{k} \varphi_i^{(k)}(x)\Big(-\Delta + m_0^2\Big)\varphi_i^{(k)}(x) + \frac{1}{4}\sum_{i,j=1}^{k} g_{ij}\varphi_i^2(x)\varphi_j^2(x) + \frac{\rho_0}{6}\sum_{i=1}^{k}\varphi_i^6(x)\right]. \tag{25}$$

Using that $\varphi_i^{(k)}(x) = \varphi_j^{(k)}(x)$, the symmetric coupling constants g_{ij} are given by $g_{ij} = (\lambda_0\delta_{ij} - \sigma^2)$. The effective Hamiltonian is written as

$$H_{eff}\left(\varphi_i^{(k)}\right) = \int d^d x \sum_{i=1}^{k}\left[\frac{1}{2}\varphi_i^{(k)}(x)\Big(-\Delta + m_0^2\Big)\varphi_i^{(k)}(x) + \frac{1}{4}(\lambda_0 - k\sigma^2)\left(\varphi_i^{(k)}(x)\right)^4 + \frac{\rho_0}{6}\left(\varphi_i^{(k)}(x)\right)^6\right]. \tag{26}$$

First-order phase transition



The effective Hamiltonian, $H_{eff}(\varphi_i^{(k)})$ in the case of additive quenched disorder is written as

$$H_{eff}\left(\varphi_i^{(k)}\right) = \int d^dx \left[\frac{1}{2}\sum_{i=1}^{k}\varphi_i^{(k)}(x)\Big(-\Delta + m_0^2\Big)\varphi_i^{(k)}(x) + \frac{\lambda_0}{4}\left(\varphi_i^{(k)}(x)\right)^4 - \frac{\sigma^2}{2}\sum_{i,j=1}^{k}\varphi_i^{(k)}(x)\varphi_j^{(k)}(x)\right]. \quad (27)$$

Using that $\varphi_i^{(k)}(x) = \varphi_j^{(k)}(x)$, the effective Hamiltonian is written as

$$H_{eff}\left(\varphi_i^{(k)}\right) = \int d^dx \sum_{i=1}^{k}\left[\frac{1}{2}\varphi_i^{(k)}(x)\Big(-\Delta + m_0^2 - k\sigma^2\Big)\varphi_i^{(k)}(x) + \frac{\lambda_0}{4}\left(\varphi_i^{(k)}(x)\right)^4\right]. \quad (28)$$

- "Disorder Effects in Dynamical Restoration of Spontaneously Broken Continuous Symmetry", G. Heymans, N. F. Svaiter and G. Krein, arXiv:2208.04445 (hep-th).
- Euclidean quantum $O(N)$ model with $N = 2$ in a continuous broken symmetry phase at low temperature with quenched disorder linearly coupled to the scalar field.
- An average over the ensemble of all realizations of the disorder is performed. We represent the quenched free energy in a series over the moments of the partition function.
- In one-loop approximation, one can prove that there is a denumerable collection of moments that can develop critical behaviour.
- Below the critical temperature of the pure system, with the bulk in the ordered phase, there are a large number of critical temperatures which take each of these moments from an ordered to a disordered phase.

- In Euclidean quantum gravity, for metrics with Euclidean signature, it is possible to have fluctuations with change of the topology.
- We discuss topological fluctuations in a Euclidean space $\mathcal{M}^d$. Using results of statistical mechanics of anisotropic disordered systems, the contribution of wormholes in Euclidean gravitational functional integral arises from a quenched randomness inherent in the $\mathbb{R} \times \mathcal{M}^{\mathrm{d}}$ manifold.
- Taking the average over all the realizations of the all disorder fields, the free-energy of the system must be calculated.
- To calculate this quantity, there are different methods in the literature, as for example the replica method, the supersymmetric and the dynamic approaches. Here we use the distributional zeta-function method.

Suppose a compact manifold of Riemannian signature $\mathcal{M}$. Let S be an action functional of the matter and gravitational fields. The partition function is

$$Z = \int [dg][d\phi] \exp\left[-S(\phi) - S(g)\right]. \tag{29}$$

For simplicity let us discuss only a neutral scalar field. The action functional is

$$S(\phi) = S_0(\phi) + S_I(\phi), \tag{30}$$

where

$$S_0(\phi) = \int d^{\mathrm{d}}x\sqrt{g}\,\frac{1}{2}\phi(x)\left(-\Delta + m^2\right)\phi(x), \tag{31}$$

and

$$S_I(\phi) = \int d^{\mathrm{d}}x\sqrt{g}\,\frac{\lambda_0}{4}\phi^4(x). \tag{32}$$

$$S(g) = -\frac{1}{16\pi G}\int_{\mathcal{M}} d^{\mathrm{d}}x\sqrt{g}(R-2\Lambda) - \frac{1}{8\pi G}\int_{\partial\mathcal{M}} Kd^{\mathrm{d}-1}\Sigma + C. \quad (33)$$

The Δ is the Laplace-Beltrami operator, $g = \det(g_{ij})$, G the Newton's constant, R is the Ricci-scalar, Λ is the cosmological constant, K is the trace of the second fundamental form on the boundary and C is a constant.

The effects of wormholes in the partition function can be written as:

$$Z = \int [dg][d\phi] \exp\left[-S(\phi,g) + \frac{1}{2}\int d^{\mathrm{d}}x \int d^{\mathrm{d}}y \sum_{i,j} C_{ij}\phi_i(x)\phi_j(y)\right]. \quad (34)$$

The last term of the action is a non-local contribution common in quenched disordered systems at low temperatures or systems with anisotropic quenched disorder. In such systems one is interested to obtain the quenched free energy.

The model which we will propose relies on an holographic principle. Our main idea is introduce a extra dimension and an additive disorder field. After the average of ensembles be taken, the contribution of the Euclidean wormholes effective action is naturally raised.

To start, let us consider a (d+1) dimensional field theory with disorder defined in the space $\mathbb{R}\times\mathcal{M}^{\mathrm{d}}$. For the scalar case, the action becomes

$$S(\phi,h,j) = \int d\sigma \int d^{\mathrm{d}}x\sqrt{g}\,\Big[\tfrac{1}{2}\phi(x,\sigma)(-\Delta+m^2)\phi(x,\sigma)+\tfrac{\lambda}{4}\phi^4(x,\sigma) + h(x,\sigma)\phi(x,\sigma)+j(x,\sigma)\phi(x,\sigma)\Big]. \quad (35)$$

Once we have a disordered theory we are free to impose a suitable covariance. We will choose a covariance:

$$\mathbb{E}[h(x,\sigma)h(y,\sigma')] = \varrho^2\delta(\sigma-\sigma')F(x-y) \quad (36)$$

$$S_{\text{eff}}\left(\phi_i^{(k)}, j_i^{(k)}\right) = \int d\sigma \int d^{\text{d}}x\sqrt{g}\Bigg\{\sum_{i=1}^{k}\left[\frac{1}{2}\phi_i^{(k)}(x,\sigma)\left(-\Delta+m^2\right)\phi_i^{(k)}(x,\sigma)\right.$$
$$\left.+\frac{\lambda}{4}\left[\phi_i^{(k)}(x,\sigma)\right]^4\right]\Bigg\} - \int d\sigma \int d^{\text{d}}x\sqrt{g}\sum_{i,j=1}^{k}\phi_i^{(k)}(x,\sigma)j_j^k(x,\sigma)$$
$$-\frac{\varrho^2}{2}\int d\sigma \int d^{\text{d}}y \int d^{\text{d}}x\; g\sum_{i,j=1}^{k}F(x-y)\phi_i^{(k)}(x,\sigma)\phi_j^{(k)}(y,\sigma). \qquad (37)$$

Such effective action has non-local contributions. To go further we can make a choice over the functional form of the multiplets $\phi_i^{(k)}(x,\sigma)$.

In a series of previous works the choice of all fields equals in the same multiplet have been made. It was found the non trivial free-energy landscape typical of complex systems and others non-trivial quantities.

Here we will make no specific choice about the fields which belongs to $k^{\rm th}$ multiplet. All the fields in the same multiplet are allowed to be different between themselves. One can show that

$$\mathbb{E}\left[Z^{k}(j,h)\right] = \int \prod_{a=2}^{k} \left[d\phi_a^{(k)}\right] [d\varphi] \exp\left[S^{I}_{CA}\left(\phi_a^{(k)}, j^{(k)}\right) + S^{(k)I}_{\varrho}(\varphi, j)\right], \tag{38}$$

where we define

$$S^{I}_{CA}\left(\phi_a^{(k)}, j^{(k)}\right) = \int d\sigma \int d^{\rm d}x \sqrt{g} \sum_{a=2}^{k} \left\{ \left[\frac{1}{2}\phi_a^{(k)}(x,\sigma)\left(-\Delta + m^2\right)\phi_a^{(k)}(x,\sigma)\right] + \frac{\lambda}{4}\left[\phi_a^{(k)}(x,\sigma)\right]^4 \right\} - \int d\sigma \int d^{d}x \sqrt{g} \sum_{a,b=2}^{k} \phi_a^{(k)}(x,\sigma) j_b^{(k)}(x,\sigma)$$

and

$$S_{\varrho}^{(k)I}\left(\varphi,F\right)=\int d\sigma\int d^{\mathrm{d}}x\sqrt{g}\left\{\left[\tfrac{1}{2}\varphi(x,\sigma)\left(-\Delta+m^2\right)\varphi(x,\sigma)\right]+\tfrac{\lambda}{4}\varphi^4(x,\sigma)\right\}$$
$$-\frac{k\varrho^2}{2}\int d\sigma\int d^{\mathrm{d}}x\sqrt{g}\int d^{\mathrm{d}}y\sqrt{g}F(x-y)\varphi(x,\sigma)\varphi(y,\sigma). \qquad (39)$$

One possible choice is the following: for $k=1$ we have the singlet, fixed by the initial configurations of fields: $\phi^{(1)}=\phi_1$. The next term, $k=2$, will be $\phi^{(2)}=\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$, where ϕ_2 is independent of ϕ_1 but the doublet carry information about ϕ_1. Such procedure lead us to

$$\phi^{(k)}=\begin{pmatrix}\phi_1\\ \phi_2\\ \vdots\\ \phi_k\end{pmatrix}, \qquad (40)$$

where all components, $\phi_i^{(k)}$, are different but $\phi^{(k)}$ contains the initial information.

- Quantum field theory on gravitational background space-time is an intermediate step toward to our understanding of the program of quantization of the gravitational field.
- Analogous systems which reproduce major features of black holes, as for example phonons in Bose-Einstein condensate. One can uses the Gross-Pitaevskii mean-field equation to discuss the Hawking effect in the condensate.
- Fluctuations of the geometry of spacetime caused by a bath of gravitons in a squeezed state has the effect of smearing the light-cone.
- It is possible to discuss an analog model for light-cone fluctuations: disorder medium with random classical fluctuations in the reciprocal of the bulk modulus.

- In Euclidean quantum gravity, it is possible to have fluctuations with change of the topology, as for example wormholes. The effects of the wormholes appears as a nonlocal contribution to the Euclidean action.
- Recently it was introduced a new technique for computing the average free energy of classical and quantum systems with quenched randomness: the distributional zeta-function method.
- Anisotropic quenched disorder introduces nonlocal contribution to the effective action in classical field theory. In Euclidean quantum field theory at low temperatures with disorder, the nonlocal contribution also appears.
- An analog model for Euclidean wormholes effects can be discussed: quenched additive anisotropic disorder in Euclidean scalar field theory.
- Work-in-progress discussing two-dimensional models.

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